SAIVA BHANU KSHATRIYA COLLEGE

(Aruppukottai Nadargal Uravinmurai Pothu Abi Viruthi Trustuku Pathiyapattathu)

ARUPPUKOTTAI

DEPARTMENT OF MATHEMATICS

QUESTION BANK

Name of the Department :	B.Sc., MATHEMATICS		
Semester (UG - III & V; PG - III) :	V	Subject Code :	SMTJC52
Name of the Subject :	REAL ANALYSIS		

Section A (Multiple Choice Questions)

Unit I:

1. Which of the following has range $(1, \infty)$?
   (a) $\log x$ on $(0, \infty)$ (b) $\frac{1}{X}$ on $(0, \infty)$ (c) e^x on $(0, \infty)$ (d) e^{-x} on $(0, \infty)$
2. Which of the following is not a closed set?
   (a) $\{x \in \mathbb{R}: 1 \leq x \leq 10\}$ (b) $\mathbb{N} \cup \{0\}$ (c) $\left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}$ (d) $\left\{m + \frac{1}{n} : m, n \in \mathbb{N}\right\}$
3. The function $f(x) = \frac{Sin\ x}{x}, (x \in R', x \neq 0)$ is ______________.
   (a) defined at $x = 0$ (b) continuous at $x = 0$
   (c) not continuous at $x = 0$ (d) $\lim_{n\to\infty} \frac{\sin x}{x}$ does not exist
4. In a metric space M, a subset E is closed if
   (a) $E \neq \bar{E}$ (b) $E = \bar{E}$ (c) $E \subset \bar{E}$ (d) $E \neq \phi$
5. Every subset of ———————— is open.
   (a) R (b) [0,1] (c) (0,1) (d) R_d

Unit II:

6. What is the diameter of the set $\{(x, y): x + y = 1\}$?
   (a) $\sqrt{2}$ (b) 2 (c) 1 (d) ∞
7. Which of the following set in R^2 is open?
   (a) $\{(x, y)/ x + y = 1\}$ (b) $\{(x, y)/ x$ and y are rational$\}$
   (c) $\{(x, y)/ x + y > 1\}$ (d) $\{(x, y)/ x2 + y2 = 1\}$
8. If $M = [0,1]$ with $d(x, y) = |x - y|$ then $B(0, \frac{1}{2})$
   (a) $(0, \frac{1}{2})$ (b) $[0, \frac{1}{2})$ (c) $[0, \frac{1}{2}]$ (d) $[0,1]$
9. If A_1 and A_2 are connected, then $A_1 \cup A_2$ is connected is
   (a) $A_1 \cap A_2 \neq \phi$ (b) $A_1 \cap A_2 = \phi$ (c) $A_1 \cup A_2 \neq \phi$ (d) $A_1 \cup A_2 = \phi$
10. If A is not bounded, then dim A □ ————————.
    (a) 1 (b) 2 (c) ∞ (d) $-\infty$

Unit III:

11. If f is continuous on [a,b], then f _________________ at [a,b].
    (a) attains maximum and minimum (b) attains maximum but not minimum

(c) attains minimum but not maximum (d) attains neither maximum nor minimum

12. Which of the following is a compact set in R?
(a) (0,1) (b) (0,1] (c) $(0,\infty)$ (d) [0,1]

13. The metric space $< M,\rho$) is compact if it is complete and ___________
(a) continuous (b) open (c) totally-bounded (d) connected

14. Which of the following is uniformly continuous?
(a) $\{f(x) = x^2,\ f:[0,1] \to R\}$ (b) $f:(0,1) \to R\,, f(x) = \frac{1}{x}$
(b) (c) $f: R \to R, f(x) = x^2$ (d) (c) $f: R \to R, f(x) = x^3$

15. What is the set $\bigcap_{n=1}^{\infty}[\frac{-1}{n}, \frac{1}{n}]$?
(a) $\{0\}$ (b) $(-1,1)$ (c) $[-1,1]$ (d) $(-\infty,\infty)$

Unit IV:

16. Measure of $\{x_1, x_2, \ldots, x_n\}$ in R is ————————
(a) 1 (b) 0 (c) 0 (d) ∞

17. The set of all rational numbers is of measure ________________.
(a) 3 (b) 2 (c) 1 (d) 0

18. $M[f:g]$ is defined by
(a) $l.u.b_{x\in g}\ f(x)$ (b) $l.u.b_{x\in g}\ f'(x)$ (c) $g.l.b_{x\in g}\ f(x)$ (d) $g.l.b_{x\in g}\ f'(x)$

19. What is the measure of a finite set A ?
(a) The cardinality $|A|$ of A (b) 0 (c) 2^k, $where\ k = |A|$ (d) None of these

20. What is the value of $U[f,\sigma]$ where $f(x) = x$ in [0,1] and $\sigma = \{0, {}^1/_3, {}^2/_3, 1\}$
(a) ${}^2/_3$ (b) ${}^1/_3$ (c) 1 (d) 0

Unit V:

21. If $f'(x) = g'(x), \forall\, x \in [0,1]$ then ___________ $= c, c \in R$.
(a) $f(x)$ (b) $f(x) - g(x)$ (c) $f(x) + g(x)$ (d) $g(x)$

22. $\lim_{x\to c} \frac{f^{n-1}(x) - f^{n-1}(c)}{x-c} =$
(a) $f^{n-1}(c)$ (b) $f^n(c)$ (c) $f^n(x)$ (d) $f^{n-1}(x)$

23. The derivative of $f(x) = \sin(x^2)$ in $\mathbb{R}$ is
(a) $\cos(x^2)$ (b)) $2x\cos(x^2)$ (c) $\sin 2x$ ` (d) $2x\sin(x^2)$

24. Let $f(x) = x$ and $g(x) = x^2$ on [a,b]. What is the value of c for which $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$
(a) 1 (b) 0 (c) ${}^1/_2$ (d) ${}^1/_3$

25. $f(x) = \begin{cases} x & x\ is\ rational \\ \sin x & x\ is\ irrational \end{cases}$ then $f'(0) =$
(a) -1 (b) 0 (c) 1 (d) > 1

Section B (7 mark Questions)

Unit I:

26. Prove that the composition of two continuous functions is again a continuous function.
27. If G_1 and G_2 are open subsets of a metric space M, then prove that $G_1 \cap G_2$ is also a open set.
28. Prove that x is a limit point of a set E in a metric space M if and only if for every open ball $B(x;r)$ about x contains at least one point of E.
29. Define open and closed sets in a metric space. Give examples to each.
30. Prove that the set R' is of second category.

Unit II:

31. Prove that a subset A of R' is connected $\Leftrightarrow$ whenever $a \in A,\ b \in A,\ a \leq b$ then $c \in A$, for any c such that $a < c < b$.
32. Prove that l^2 is complete.
33. Prove that every totally bounded subset of a metric space is bounded.
34. Let A be a subset of $\mathbb{R}$ with the property that whenever $a, b \in A, (a.b) \subseteq A$, then prove that A is connected.
35. If $< M, \rho >$ is a complete metric space and A is a closed subset of M, then prove that $< A, \rho >$ is also complete.

Unit III:

36. If M is a compact metric space, then prove that M has Heine-Borel property.
37. Prove that the continuous image of a compact metric space is compact.
38. If every sequence of points in a metric space M has subsequence converging to a point in M, then show that M is compact.
39. Prove that a real valued continuous function f on a compact metric space M attains its maximum value at some point of the domain M.
40. Prove that any compact set in a metric space is closed.

Unit IV:

41. If f is a continuous function on [a,b] and σ and τ are two subdivisions of [a,b], then prove that $U[f,\sigma] \geq L[f,\tau]$.
42. If f is a bounded function on the closed bounded interval [a,b], then prove that $f \in \mathcal{R}[a,b] \Leftrightarrow$ for each $\epsilon > 0, \exists\ \sigma$, a subdivision of [a,b] such that $U[f;\sigma] < L[f:\sigma] + \varepsilon$.
43. If $f \in \mathcal{R}[a,b]$ and λ is any real number, then show that $\lambda f \in \mathcal{R}[a,b]$ and also show that $\int_a^b \lambda f = \lambda \int_a^b f$.
44. If $f \in \mathcal{R}[a,b]$, then prove that $|f| \in \mathcal{R}[a,b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

45. If $f \in \mathcal{R}[a,b]$, $g \in \mathcal{R}[a,b]$ and if $f(x) \le g(x)$, almost everywhere $(a \le x \le b)$, then prove that $\int_a^b f \le \int_a^b g$.

Unit V:

46. Prove that $(fg)'(c) = f'(c)g(c) + f(c)g'(c)$
47. State and prove law of mean
48. State and prove Rolle's theorem.
49. Let f be a 1-1 real valued function on an interval J and let ϕ be the inverse function for f. If f is continuous at $c \in J$ and if ϕ has derivative at $d = f(c)$ with $\phi'(d) \neq 0$, prove that f is differentiable at c and $f'(c) = \frac{1}{\phi'(d)}$.
50. If f has a derivative at every point of [a,b], then prove that f' takes on every value between $f'(a)$ and $f'(b)$.

<u>Section C (10 mark Questions)</u>

Unit I:

51. Let M_1 and M_2 be two metric spaces. Prove the necessary and sufficient condition for a function f on M_1 to be continuous is that the inverse image $f^{-1}(V)$ of every open set V in M_2 is open in M_1.
52. If f and g are continuous functions at $a \in M$, metric space, then prove that fg and $f+g$ are continuous at a.

Unit II:

53. State and prove Picard fixed point theorem.
54. Prove that $\mathbb{R}^n$ is complete.

Unit III:

55. The metric space M is compact if and only if whenever $\mathcal{F}$ is a family of closed subsets of M with finite intersection property, then $\bigcap_{F \in \mathcal{F}} F \neq \phi$.
56. Prove that the continuous function defined on a compact metric space is uniformly continuous.

Unit IV:

57. If f is a bounded function on [a,b], then prove that $f \in \mathcal{R}[a,b] \Leftrightarrow f$ is continuous at almost every point in [a,b].
58. If $f, g \in \mathcal{R}[a,b]$, then prove that If $f + g \in \mathcal{R}[a,b]$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

Unit V:

59. State and prove fundamental theorem of calculus.
60. State and prove second fundamental theorem of calculus.

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