# SAIVA BHANU KSHATRIYA COLLEGE

**(Aruppukottai Nadargal Uravinmurai Pothu Abi Viruthi Trustuku Pathiyapattathu)**

**ARUPPUKOTTAI**

## DEPARTMENT OF MATHEMATICS
## QUESTION BANK

| | | | |
|---|---|---|---|
| Class : | B.Sc., MATHEMATICS | | |
| Semester (UG - III & V; PG - III) : | Semester V | Subject Code : | SMTJC51 |
| Name of the Subject : | MODERN ALGEBRA | | |

**<u>Section A (Multiple Choice Questions)</u>**

**Unit I**

1. Which of the following is not a group?

   (a) (Z, +) (b) (R, +) (c) (N, +) (d) (Q, +)

2. $(Z_{12}, \oplus)$, $< 3 >$ = ---------------------------

   (a) {0, 3, 6, 9} (b) {0} (c) $Z_{12}$ (d) {0, 3, 9}

3. Order of a non-zero element in $(\mathbb{Z}, +)$ is

   (a) $\infty$ (b) 0 (c) 1 (d) 2

4. The Statement $(ab^2) = a^2b^2$, $\forall$ a, b $\in$ G is true if --------------------------

   (a) G is abelian (b) G is non-abelian (c) G is any group (d) G is finite

5. Number of elements in $S_n$ = -------------------

   (a) n (b) 2n (c) 2n! (d) n!

**Unit II**

6. Let H = {0, 4} be a subgroup of G = $(Z_8, \oplus)$. Then [G H] = ----------------

   (a) 3 (b) 4 (c) 2 (d) 1

7. O $(Z_6/<3>)$ is ----------------------------

   (a) 6 (b) 5 (c) 4 (d) 3

8. A group of order 24 cannot have a group of order------------------------

   (a) 3 (b) 2 (c) 8 (d) 5

9. The Index of {0, 4} in $(Z_8, \oplus)$ is ----------------------

   (a) 0 (b) 4 (c) 8 (d) 2

10. Let H be a subgroup of group G. Let R denote number of right Cosets of H in G, and L denote number of left Cosets of H in G. Then----------------------

    (a) $R < L$ (b) $L < R$ (c) $R = L$ (d) $R \neq L$

**Unit III**

11. Let $f : G \to G^{|}$ be an isomorphism. If G is abelian then $G^{|}$ is --------------------------
 (a) abelian (b) not abelian (c) need not be abelian (d) none of these
12. The number of automorphisms of acyclic group of order n is------------------------
 (a) P (n) (b) n (c) n - 1 (d) ∅ (n)
13. The Kernel of the map $f : (\mathbb{Z}, +) \to (\mathbb{R}^*, \cdot)$ is given by $f(x) = 3^x$ is -----------------------
 (a) 1 (b) 0 (c) 3 (d) none of these
14. $f : (\mathbb{Z}, +) \to (\mathbb{C}^*, \cdot)$ is given by $f(n) = i^n$ is ------------------
 (a) one-one (b) onto (c) homomorphism (d) isomorphism
15. Any infinite cyclic group is isomorphic to ---------------------------
 (a) $(Z_n, \oplus)$ (b) (Z, X) (c) (Z, +) (d) (R, +)

**Unit IV**

16. If------------------, then $Z_n$ is a field.
 (a) n = 4 (b) n = 6 (c) n = 8 (d) n is prime
17. Characteristic of ring $Z_n$ is ---------------------
 (a) n + 1 (b) n - 1 (c) n (d) n + 2
18. $(\rho(s), \Delta, n)$ is a -----------------
 (a) Commutative ring (b) Boolean ring (c) commutative ring with identity (d) All of these.
19. The characteristic of any field is -----------------------
 (a) 0 (b) prime (c) 0 (or) Prime (d) Neither 0 nor Prime
20. The characteristic of any Boolean ring is -----------------------
 (a) 0 (b) 2 (c) 1 (d) ∞

**Unit V**

21. Let I be an ideal of R with 1∈ I. Then -----------------

    (a) I = ∅ (b) I ≠ R (c) I = R (d) None

22. If --------------------, (n) is a maximal ideal in Z.

    (a) n = 4 (b) n = 6 (c) n = 8 (d) n is prime

23. The only ideals of a field F are --------------------

    (a) {0} (b) F (c) {0} or F (d) {0} & F

24. {0} is a -------------------ideal of Z.

    (a) prime (b) maximal (c) prime but not maximal (d) maximal but not prime

25. The maximal ideal of Z is --------------------------

    (a) (0) (b) (6) (c) (4) (d) (2)

**<u>Section B (7 mark Questions)</u>**

**Unit I**

26. Prove that any permutation can be expressed as a product of disjoint cycles.
27. Prove that the union of two subgroups of a group is again a subgroup if one is contained in the other.
28. Let G be a group in which every elements is of finite order. Whether the group G is finite? Identify.
29. Prove that a subgroup of a cyclic group is cyclic
30. Define abelian group. Prove that ($\rho$(s), Δ) is an abelian group.

**Unit II**

31. Define Coset. State and prove Euler's theorem.
32. Prove that every group of prime order is cyclic.
33. Prove that the quotient group G / N is a group.
34. Define normal subgroup. Prove that a subgroup N of G is normal ⟺ the product of two right cosets of N is again a right coset of N.

35. Show that if H and N are subgroups of a group G and N is normal in G, then H ∩ N is normal in H. Show by an example that H ∩ N need not be normal in G.

**Unit III**

36. Show that $(\mathbb{R}^*, \cdot)$ is not isomorphic to $(\mathbb{R}^*, +)$ .
37. Define isomorphism of a group. Let $G = \left\{\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}\right\}$ and $f: G \to \mathbb{R}^*$,

    $f\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} = a$. Then $G \cong \mathbb{R}^*$. Prove this.
38. Prove that the number of automorphisms of a cyclic group of order n is ∅ (N)
39. If G is a group and H is the Centre of G, then show that G / H ≃ I(G)
40. Let $f: G \to G'$ be a homomorphism. Then prove that f is 1 – 1 ⟺ ker f = {e}.

**Unit IV**

41. Prove that the characteristics of an integral domain is either 0 or a prime number.
42. Construct the Cayley table for the ring $(\rho(s), \Delta$, n) where S = {1, 2}
43. Prove that a finite commutative ring R without zero divisors is a field.
44. Prove: A non-empty subset S of a ring R is a sub ring ⟺ a b ∈ S & a – b ∈ S ∀ a, b ∈ S.
45. Prove that any finite integral domain is a field.

**Unit V**

46. Let R be a commutative ring with identity. Then prove that R is a field ⟺ R has no proper ideals.
47. Let R be any commutative ring with identity. Let P be an ideal of R. Then prove that P is a prime ideal ⟺ $\frac{R}{P}$ is an integral domain.
48. Define Maximal ideal. Let p be any prime. Then prove that (p) is maximal ideal in Z.
49. If $F'$ is any field containing the integral domain D, then prove that $F'$ contains a subfield which is isomorphic to a field F.
50. Let R and R/ be ring and $f: R \to R'$ be a homomorphism. If S is an ideal of R, then prove that f(S) is an ideal of f(R).

**<u>Section C (10 mark Questions)</u>**

**Unit I**

51. Let A and B be two subgroups of a group G.

Then prove that AB is a subgroup of G $\Leftrightarrow$ AB = BA

52. (a). Prove that ($Z_8$, $\oplus$ ) is cyclic and find all its generators.

(b) If $\alpha = (1\ 2\ 3\ 4)$, $\beta = (1\ 3)\ (1\ 4)$, then find $\alpha\beta, \beta\alpha, \alpha^{-1}, \beta^{-1}, \alpha^{-1}\beta^{-1}$.

**Unit II**

53. State and prove Lagrange's theorem on groups.
54. Let N be subset of G. Then prove that the following are equivalent.

(a). N is a normal subgroup of G.

(b). $\alpha$ N $\alpha^{-1}$ = N, for all $\alpha \in$ G

(c). $\alpha$ N $\alpha^{-1} \subseteq$ N, for all $\alpha \in$ G

(d). $\alpha$ n $\alpha^{-1} \in$ N, for all $n \in$ G

**Unit III:**

55. State and prove Cayley's theorem.
56. State and prove the fundamental theorem of homomorphism in groups.

**Unit IV**

57. Prove that every finite integral domain is a field.
58. Prove that $\mathbb{Z}_n$ is an integral domain iff n is prime.

**Unit V**

59. Prove that any integral domain can be embedded in a field
60. Let R be a commutative ring with identity. An ideal M of R is maximal iff $\frac{R}{M}$ is a field.