SAIVA BHANU KSHATRIYA COLLEGE

(Aruppukottai Nadargal Uravinmurai Pothu Abi Viruthi Trustuku Pathiyapattathu)

ARUPPUKOTTAI

DEPARTMENT OF MATHEMATICS

QUESTION BANK

Class:	B.Sc., Mathematics		
Semester (UG - III & V; PG - III) :	UG – V	Subject Code :	SMTJA51
Name of the Subject :	Graph Theory		

Section A (Multiple Choice Questions)

Unit I:

1. A connected (p, q) graph contains a cycle iff ________
   (a) $q < p$ (b) $q \neq p$ (c) $q \geq p$ (d) none
2. A graph is said to be k-regular if degree of every vertex is ________
   (a) $k + 1$ (b) k (c) $k - 1$ (d) $k + 2$
3. The number of odd degree vertices in a graph G is _____
   (a) odd (b) even (c) both odd & even (d) none
4. Empty graph is also known as ________
   (a) Trivial graph (b) Regular graph (c) Bipartite graph (d) none
5. If the origin and terminus to walk are same, the walk is known as ________
   (a) open (b) path (c) closed (d) none

Unit II:

6. A Hamilton cycle has ________
   (a) all vertices (b) some vertices (c) all edges (d) some edges
7. An Euler graph has no vertex of ________ degree
   (a) even (b) odd (c) prime (d)composite
8. An Eulerian trail has _____
   (a) all vertices (b) some vertices (c) all edges (d) some edges
9. A connected graph G there is an eulerian trail iff the number of vertices of odd degree is _
   (a) zero (b) one (c) two (d) either zero or two
10. Every Hamiltonian graph is ________
    (a) connected (b) non connected (c) 2-connected (d) minimally connected

Unit III:

11. Every tree has ________
    (a) 1 center (b) 2 center (c) 1 or 2 centers (d) 1 and 2 centers
12. Any tree with n vertices has ________ edges.
    (a) n (b) $n + 1$ (c) $n - 1$ (d) $\frac{n}{2}$
13. The diagonal entries of an adjacency matrix are ________
    (a) 1 (b) 2 (c) zeros (d) none
14. A (p, q)-graph G is a bipartite graph iff it contains no _____ cycles
    (a) odd (b) even (c) both odd & even (d) none
15. Adjacent edges are called _____ edges
    (a) dependent (b) independent (c) parallel (d) none

Unit IV:

16. K_5 is ________
    (a) Planar (b) Non-Planar (c) Euler graph (d) none
17. Euler polyhedron formula is ________

(a) $V - E + F = 2$ (b) $V + E - F = 2$ (c) $V - E + F - 1$ (d) $V + E - F = 0$

18. $K_{3,3}$ is _________
    (a) Planar (b) Non-Planar (c) Euler graph (d) none
19. If G is a (p, q) planar graph then $\delta(G)$ ______
    (a) $= 6$ (b) $= 7$ (c) ≤ 5 (d) ≤ 3
20. Every subdivision of a nonplanar graph is ______
    (a) planar (b) nonplanar (c) polyhedron (d) regular polyhedron

Unit V:

21. The chromatic number of K_p is _________
    (a) 0 (b) $p - 1$ (c) p (d) $p - 2$
22. A digraph is called disconnected if the underlying graph is_________
    (a) connected (b) disconnected (c) strongly connected (d) weekly connected
23. If H is a subgraph of a graph G, then _________
    (a) $\chi(G) \leq \chi(H)$ (b) $\chi(G) \geq \chi(H)$ (c) $\chi(G) = \chi(H)$ (d) $\chi(G) > \chi(H)$
24. If χ is the chromatic number, then $\chi(C_{2n+1}) =$ ______
    (a) 1 (b) 2 (c) 3 (d) 4
25. (p, q) digraph refers p vertices and q ______
    (a) edges (b) arcs (c) cycles (d) paths

Section B (7 mark Questions)

Unit I:

26. State and prove Hand Shaking theorem.
27. Prove that the number of odd degree vertices in any graph is always even.
28. If $q \geq p$, then prove that (p, q) graph contains a cycle.
29. Let G be a graph on atleast 6 vertices then prove that G and $\overline{G}$ contains a triangle.
30. Prove that every nontrivial graph contains at least two vertices which are not cut-vertices.

Unit II:

31. State and prove Fleury's algorithm.
32. Prove that in a connected graph G, there is an Eulerian trial iff the number of vertices of odd degree is either zero or two.
33. If G is a Hamiltonian graph and $S \neq \phi$, $S \subseteq V(G)$ then prove that $\omega(G - S) \leq |S|$.
34. Prove that if G is Eulerian then every point of G has even degree.
35. Prove that G is a graph with $n \geq 3$ and $m \geq \frac{n^2-3n+6}{2}$ then G is Hamilton.

Unit III:

36. Prove that a graph G is a tree if and only if every two vertices of G are connected by a unique path.
37. State R.C. Prim's algorithm with example.
38. Prove that every connected graph G contains a spanning tree.
39. Explain J.B. Kruskal's algorithm with example.
40. Prove that a connected (p, q)-graph is a tree iff every edge is a cut-edge.

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Unit IV:

41. State and prove Euler formula for planar graph.
42. Prove that a graph G on p vertices is connected iff $(A+I)^{p-1}$ has no zero entries.
43. State and prove Maclane theorem.
44. If G is a plane (p,q)-graphwith $\delta(G) \geq 3$, then prove there is a face in G of degree ≤ 5.
45. State and prove Wagner theorem.

Unit V:

46. If G is a (p,q)-graph, then prove that $\chi(G) \geq \frac{p^2}{p^2-2q}$.
47. Let G be a graph and let u and v be non-adjacent vertices in G then prove that $\chi(G) = min\{\chi(G+(u,v)), \chi(G.uv)\}$.
48. Show that for any graph G, $\chi(G) \leq \Delta(G)+1$.
49. If G is a bipartite graph with $q(G) \geq 1$, then prove that $\chi_1(G) = \Delta(G)$.
50. Write an algorithm for vertex colouring of a graph.

<u>Section C (10 mark Questions)</u>

Unit I:

51. If $q > \frac{p^2}{4}$, then prove that every (p,q)-graph contains a triangle.
52. For any Graph G, prove that $q(G) \geq p(G) - \omega(G)$.

Unit II:

53. Prove that a nontrivial connected graph is Eulerianiff it has no vertex of odd degree.
54. Prove that any trail constructed by Fleury's algorithm is a closed Eulerian trail in G.

Unit III:

55. State and prove Hall's theorem.
56. Prove that a (p,q)-graph G is a bipartite graph iff it contains no odd cycle.

Unit IV:

57. Explain there are exactly five regular polyhedral.
58. Prove that (i) if H is a subgraph of a graph G, then $\chi(G) \geq \chi(H)$

    (ii) if G is a (p,q)-graph, then $\chi(G) \geq \frac{p^2}{p^2-2q}$.

Unit V:

59. Prove that for any given integer $k(\geq 1)$ there exists a triangle free graph with chromatic number k.
60. Prove that if G is a graph on p vertices, then

    a) $2\sqrt{p} \leq \chi(G) + \chi(\overline{G}) \leq p+1$ b) $p \leq \chi(G)\chi(\overline{G}) \leq \frac{(p+1)^2}{4}$.