SAIVA BHANU KSHATRIYA COLLEGE

(Aruppukottai Nadargal Uravinmurai Pothu Abi Viruthi Trustuku Pathiyapattathu)

ARUPPUKOTTAI

DEPARTMENT OF MATHEMATICS

QUESTION BANK

Class :	B.Sc., Mathematics		
Semester (UG - III & V; PG - III) :	UG - III	Subject Code :	SMTJS51
Name of the Subject :	FOURIER SERIES AND LAPLACE TRANSFORM		

Section A (Multiple Choice Questions)

Unit I:

1. A function $f(x)$ is even function if _____
   (a) $f(x) = f(x^2)$ (b) $f(x) = f(-x^2)$ (c) $f(x) = f(-x)$ (d) $f(x) = -f(-x)$
2. A function $f(x)$ is odd function if _____
   (a) $f(x) = f(x^2)$ (b) $f(x) = f(-x^2)$ (c) $f(x) = f(-x)$ (d) $f(x) = -f(-x)$
3. An example of an odd function is _____
   (a) $x\ sinx$ (b) $x^2 cosx$ (c) $x^2\ sinx$ (d) $sinx + cosx$
4. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx$ is _____
   (a) $sinx$ (b) $cosine$ (c) tan (d) cot
5. $f(x) = \sum_{n=1}^{\infty} b_n sinnx$ is _____
   (a) $sinx$ (b) $cosine$ (c) tan (d) cot

Unit II:

6. The value of $'b_n'$ for the function $f(x) = x, -l < x < l$ is _____
   (a) 0 (b) π (c) $\frac{2(-1)^{n+1}l}{n\pi}$ (d) $\frac{2l}{n\pi}$
7. The value of $'a_0'$ in the Fourier cosine series expansion of $f(x) = \pi - x$ in $(0, \pi)$ is ____
   (a) $\frac{\pi}{2}$ (b) 0 (c) π (d) 2π
8. The period of $sinx$ is _____
   (a) 6π (b) 4π (c) 2π (d) π
9. If $f(x) = x^2$ in $-\pi < x < \pi$, then the value of b_n is _____
   (a) 0 (b) $\frac{2\pi^3}{3}$ (c) $\frac{\pi^3}{3}$ (d) $\frac{4\pi^3}{3}$
10. If $m = n$ then $\int_0^{\pi} cosmx\ cosnx\ dx$ = _____
    (a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π

Unit III:

11. L (e^{-t}) = _____
    (a) $\frac{1}{s+1}$ (b) $\frac{1}{s-1}$ (c) $\frac{c}{s+1}$ (d) $\frac{1}{s^2}$
12. L(F(at)) = _____
    (a) F(as) (b) aF(s) (c) $\frac{1}{a}F\left(\frac{S}{a}\right)$ (d) F $\left(s/a\right)$
13. L(f(t)) = F(s) then l(t^2f(t)) = _____
    (a) $\frac{s}{s+a}$ (b) e^{-2t} (c) $(-1)\frac{d}{ds}F(s)$ (d) $\frac{d^2}{ds^2}F(s)$

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14. L(sinat) = _____

(a) $\frac{a}{s^2+a^2}$ (b) $\frac{s}{s^2+a^2}$ (c) $\frac{a}{s^2-a^2}$ (d) $\frac{s}{s^2+a^2}$

15. L(t) = ____

(a) 1 (b) $\frac{1}{s^2}$ (c) $\frac{1}{s}$ (d) $\frac{2}{s^3}$

Unit IV:

16. $L^{-1}(\cos ax)$ = _____

(a) $\frac{s}{s^2+a^2}$ (b) $\frac{a}{s^2+a^2}$ (c) $\frac{a}{s^2-a^2}$ (d) $\frac{s}{s^2-a^2}$

17. $L^{-1}(1)$ = _____

(a) 1 (b) 0 (c) s (d) $\frac{1}{s}$

18. $L^{-1}(F(s+a))$ = _____

(a) $L^{-1}(F(s))$ (b) $e^{-at} L^{-1}(F(s))$ (c) $e^{at} L^{-1}(F(s))$ (d) $L^{-1}(F'(s))$

19. $L^{-1}\left(\frac{1}{S+3}\right)$ = _____

(a) sint (b) sin3t (c) cos3t (d) cos2t

20. $L^{-1}\left(\frac{1}{S^2+1}\right)$ = _____

(a) sint (b) sin2t (c) cost (d) cos2t

Unit V:

21. $L^{-1}(F'(s))$ = _____

(a) $L^{-1}(F(s))$ (b) $-tL^{-1}(F'(s))$ (c) $L^{-1}(F(ts))$ (d) $-tL^{-1}(F(s))$

22. $L(y')$ = _____

(a) $sL(y) - y(0)$ (b) $s^2 L(y) - sy'(0)$ (c) $sL(y) - y'(0)$ (d) $s^2 L(y) - sy(0) - y'(0)$

23. $L(y'')$ = _____

(a) $sL(y) - y(0)$ (b) $s^2 L(y) - sy'(0)$ (c) $sL(y) - y'(0)$ (d) $s^2 L(y) - sy(0) - y'(0)$

24. $L^{-1}\left(\frac{2s}{(s^2+1)^2}\right)$ = _____

(a)tsint (b) tcost (c) t^2 sint (d) t^2 cost

25. $L^{-1}(sF(s))$ = ____

(a) $e^{at}F(s)$ (b) $\frac{1}{a}F\left(\frac{t}{a}\right)$ (c) $\frac{d}{dt}L^{-1}(F(s))$ (d) F(s)

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Section B (7 mark Questions)

Unit I:

26. Expand $f(x) = \begin{matrix} -x & -\pi < x < 0 \\ x & 0 < x < \pi \end{matrix}$ as Fourier series.
27. Find the Fourier constant $'a_0'$and $'a_n'$ for the function $f(x) = \begin{matrix} x & 0 < x < \pi \\ 2\pi - x & \pi < x < 2\pi \end{matrix}$.
28. Express as a Fourier series of the function $f(x) = \begin{matrix} a & 0 < x < \pi \\ -a & \pi < x < 2\pi \end{matrix}$.
29. Express Fourier expansion $f(x) = \pi^2 - x^2,\ -\pi < x < \pi$.

Find the Fourier constant $'a_0'$and $'b_n'$ for the function $y = \begin{matrix} 1+x & 0 < x < \pi \\ -1+x & -\pi < x < 0 \end{matrix}$

Unit II:

30. Find the Fourier sine series and the Fourier cosine series corresponding to the function, $f(x) = \pi - x$ when $0 < x < \pi$ defined in the interval 0 to π.
31. Express $f(x) = c - x$ where $0 < x < c$ as a half range cosine series with period $2c$.
32. Obtain a cosine series for $f(x) = e^x,\ 0 < x < \pi$.
33. Find the Fourier sine series for $f(x) = K$, in $0 < x < \pi$.
34. Expand $f(x) = x$ as sine series in the interval $(0, \pi)$.

Unit III:

35. Find: L ($\sin^3$ at)
36. Evaluate: $\int_0^\infty te^{-3t}\cos t dt$
37. Find: $L\left(\frac{1-e^t}{t}\right)$
38. Find: L(t^2 cosh at)
39. Find: $L\left(\frac{1-\cos t}{t}\right)$

Unit IV:

40. Find: $L^{-1}\left(\frac{1}{(s+3)^2+25}\right)$
41. Find: $L^{-1}\left(\frac{s}{(s+2)^2}\right)$
42. Find: $L^{-1}\left(\frac{s}{(s^2+2s+5}\right)$
43. Find: $L^{-1}\left(\frac{1}{(s^2+a^2)^2}\right)$
44. Find: $L^{-1}\left(\frac{s}{(s^2-1)^2}\right)$

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Unit V:

45. Solve using Laplace transform
$\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$ *with the conditions* $x(0)=1, y(0)=0$
46. Solve the equation $t\frac{d^2y}{dt^2} - (2+t)\frac{dy}{dt} + 3y = t - 1$ *when* $y(0) = 0$
47. Solve using Laplace transform
$\frac{dx}{dt} + 2x - 3y = t, \frac{dy}{dt} - 3x + 2y = e^{2t}$ *with the conditions* $x(0)=0, y(0)=0$
48. Solve using Laplace transform $\frac{d^2y}{dt^2} + t\frac{dy}{dt} - y = 0$ *when* $y(0)=0, y'(0)=1.$
49. Using Laplace transform, solve $y' + 3y = e^{-2x}$ *given* $y(0) = 4$

Section C (10 mark Questions)

Unit I:

50. Find the Fourier series for the function $f(x) = x^2,\ -\pi < x < \pi$ and deduce that
a) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$
b) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots = \frac{\pi^2}{12}$
c) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$
51. Find the Fourier series of the function $f(x) = \frac{1}{2}(\pi - x)$ in the interval $(0,2\pi)$.

Unit II:

52. If $f(x) = \begin{matrix} x(\pi-x) & for & 0<x<\pi \\ -\pi(\pi-x) & for & \pi<x<2\pi \end{matrix}$, then prove that $f(x) = \frac{8}{\pi}\left(\sin x + \frac{1}{3^2}\sin 3x + \right.$ $152\sin 5x + \ldots$. Deduce that $1-132+152-\ldots=\pi 332$
53. Find the Fourier cosine series and Fourier sine series for $f(x) = x$ in $(0, \pi)$.

Unit III:

54. Find: (a) $L(xe^{-x}\sin x)$ (b) $L(x^2 e^{-4x})$
55. Find: (a) $L\left(\frac{1-e^t}{t}\right)$ (b) $\int_0^\infty \frac{e^{-t} - e^{-2t}}{t} dt$

Unit IV:

56. Find: (a) $L^{-1}\left(\frac{1}{s(s+a)}\right)$ (b) $L^{-1}\left(\frac{1}{(s+1)(s^2+2s+2)}\right)$
57. Find: $L^{-1}\left(\frac{1+2s}{(s+2)^2(s-1)^2}\right)$

Unit V:

58. Solve $y'' + 4y' + 13y = 2e^{-x}$ using Laplace transform given $y(0)$ and $y'(0) = -1$

59. Solve $y'' + 2y' - 3y = \sin t$ given, $y = 0$, $y'(0) = 0$ when $t = 0$